Adaptive Quantum Coherence, Foundational Closure

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Abstract

The original Adaptive Quantum Coherence (AQC) paper established the conceptual program of coherence-regulated quantum dynamics and adaptive measurement, but left unresolved the mathematical status of state dynamics, branch weighting, measurement update, and composite-system consistency. This paper does not revisit those motivations. Its purpose is narrower and more technical, to provide the minimal formal closure required for AQC to function as a consistent finite-dimensional, nonrelativistic quantum measurement framework. To that end, I reformulate AQC at the level of density operators, distinguish global coherence as a state functional from measurement-contextual branch weighting, and define an admissible class of coherence-arbitrated dynamics compatible with effective measurement theory. I then introduce an instrument-level AQC measurement formalism in which outcome probabilities are generated by normalized contextual branch weights rather than an independent collapse postulate. Within standard projective measurement contexts, I prove an exact Born-recovery result, showing that AQC reproduces ordinary quantum probabilities in the closure regime. I further establish composite-system consistency, including no-signaling under local measurement choices, so that entangled systems remain compatible with the operational structure of standard quantum theory. The result is not a deformation of ordinary quantum mechanics, but a formal completion of the original AQC program in the regime where its mathematical gaps were most exposed. Standard quantum mechanics is recovered as the operational limit of the closure framework, while infinite-dimensional, relativistic, and field-theoretic extensions are left for future work.

Keywords: quantum foundations, measurement problem, born rule, decoherence, projective measurement, density matrices, quantum entanglement, no-signaling, quantum probabilities, interpretation of quantum mechanics, adaptive quantum coherence

Scope and Relation to the Original AQC Paper

The original Adaptive Quantum Coherence (AQC) paper introduced a framework for understanding quantum measurement as a process of adaptive coherence arbitration rather than wavefunction collapse or ontological branching. Its central proposal was that quantum evolution, especially in measurement settings, should be understood through coherence-weighted state selection, with stable outcomes arising from structured persistence rather than from an independent collapse postulate. In that sense, the original paper was not merely interpretive commentary but the opening statement of a broader program to recast quantum mechanics in coherence-first terms while preserving its empirical reach.

That initial framework, however, was deliberately more programmatic than complete. Several of the formal components required for a mathematically closed quantum theory were left open. The original paper did not provide a full density-operator formulation suitable for mixed states and open subsystems, did not specify a basis-stable branch-weight rule for measurement contexts, did not introduce an instrument-level formalism for measurement update, did not prove exact recovery of the Born rule, and did not establish a composite-system consistency theorem sufficient to secure entanglement compatibility and no-signaling. These omissions do not invalidate the program, but they leave its status formally incomplete.

The purpose of the present paper is to close those gaps in a restricted and deliberately conservative setting. This work is confined to finite-dimensional, nonrelativistic quantum systems and should be read as a closure paper rather than a deformation paper. It does not seek to introduce new experimentally distinct predictions or to replace the operational success of standard quantum mechanics. Its narrower aim is to show that Adaptive Quantum Coherence can be written as a mathematically coherent extension of standard measurement theory, one that recovers ordinary quantum statistics exactly in the closure regime while supplying the formal structures the original framework lacked.

Several larger questions therefore remain outside the scope of this paper. I do not attempt here to extend the framework to infinite-dimensional systems, formulate a relativistically covariant version, develop a quantum field theoretic generalization, or analyze regimes in

which AQC would depart nontrivially from standard quantum predictions. Those problems matter, but they are downstream of a more immediate task. Before asking whether AQC can generalize beyond ordinary quantum measurement theory, it is necessary to show that it can first close consistently within it.

Minimal Operational Core

This paper does not begin from a preferred interpretation of quantum mechanics. It begins from the smaller and more disciplined question of what any acceptable closure of the Adaptive Quantum Coherence program must preserve at the operational level. The point of proceeding this way is to narrow the task before formalizing it. A theory that aims to complete the measurement layer of quantum mechanics does not first need a maximal ontology; it first needs to recover the minimal structure by which quantum systems are actually represented, measured, and related across composite settings.

That minimal structure is non-negotiable. In the finite-dimensional setting considered here, the closure must preserve at least five features of ordinary quantum practice: the assignment of physical states to systems, the outcome statistics of standard measurements, the repeatability of ideal projective measurement, the consistency of reduced states for subsystems embedded in larger composites, and the no-signaling constraint across entangled systems under local measurement choices. These are not optional interpretive preferences. They are the operational conditions any proposed completion must satisfy exactly if it is to count as a closure of quantum measurement theory rather than as an alternative empirical model.

By contrast, collapse, ontological branching, many-worlds narratives, and related interpretive structures are not taken here as primitive elements of the operational core. They may function as explanatory overlays in one framework or another, but they are not part of any minimal empirical architecture that must be preserved. The present paper therefore does not assume them, reject them, or attempt to settle the broader metaphysical disputes to which they belong. Its narrower question is only this; what minimal additional formal

structure is required to reconstruct the measurement layer coherently within the AQC program, without losing any of the standard operational content?

This methodological restriction governs everything that follows. Every formal object introduced later, including a global coherence functional, a contextual branch-weight rule, an admissible state-update law, and an instrument-level measurement formalism, must justify itself by closing one of the operational gaps identified here. Nothing is added merely for conceptual symmetry or interpretive ambition. That discipline is essential. It keeps the paper focused on closure rather than expansion, and it prevents the framework from inheriting more theoretical burden than the present task requires.

Formal Setting and Axioms

This paper does not attempt to rebuild quantum mechanics from first principles. Its starting point is ordinary finite-dimensional quantum theory in its standard operator form. The task is to identify the smallest additional structure required to close the original AQC framework where it was formally incomplete, namely at the level of measurement, branch weighting, and effective state update. The strategy is therefore conservative. Standard quantum mechanics supplies the underlying kinematic and operational background, while AQC enters only through a coherence-arbitration layer introduced to formalize the measurement-theoretic role the earlier paper described programmatically.

Axiom 1 (State Representation). Physical states are represented by density operators

$$\rho \in \mathcal{D}(\mathcal{H})$$

where $\mathcal{H}$ is a finite-dimensional Hilbert space and $\mathcal{D}(\mathcal{H})$ denotes the set of positive trace-one operators on $\mathcal{H}$.

This choice is not merely notational. It places the theory immediately in the correct setting for mixed states, open subsystems, entangled composites, and measurement update. It also avoids the limitations of a pure state formalism, which is too narrow for a closure paper whose central task concerns measurement contexts and composite-system consistency.

Axiom 2 (Two-Level Coherence Structure). The theory contains two distinct coherence-theoretic objects. First, there exists a global coherence functional

$$C[\rho]$$

defined on physical states. Second, for each measurement context $\mathcal{M}$, and where required an effective apparatus or environmental specification $\mathcal{E}$, there exists a contextual branch-weight functional

$$W_i(\rho, \mathcal{M}, \mathcal{E}),$$

defined for the outcome channels i associated with that context.

This distinction is central. The original AQC program used coherence language to describe both intrinsic state structure and the effective selection of measurement outcomes. In a formal completion, those roles cannot remain conflated. A global state functional and a contextual branch-weight rule solve different problems and are constrained by different requirements. The former concerns the structure of states independently of a particular measurement decomposition; the latter concerns how outcome channels are weighted once a measurement interaction has made such a decomposition physically relevant.

Axiom 3 (Admissibility of Global Coherence). The global coherence functional $C[\rho]$ is bounded, basis-independent, and compatible with a well-defined composition law on product states and coarse-grainings.

The purpose of this axiom is to prevent coherence from becoming an artifact of arbitrary representation. If $C[\rho]$ is to serve as a meaningful state-level quantity, it must depend on the physical state itself rather than on a chosen basis or descriptive parametrization. At the same time, because the theory must apply to composite systems and effective descriptions at different resolutions, the functional must behave coherently under subsystem composition and benign coarse-graining. No stronger claim is imposed here. In particular, this paper does not assume in advance that the chosen functional must be additive, multiplicative, or unique. It requires only that its behavior under these operations be mathematically well-defined and physically interpretable.

Axiom 4 (Instrument-Level Measurement Structure). Measurements are represented by an AQC instrument

$$\{\mathcal{I}_i^{\mathcal{M}}\},$$

indexed by the outcome channels i of measurement context $\mathcal{M}$, such that the induced outcome probabilities are determined by normalized contextual branch weights.

This axiom makes explicit that AQC closure must occur at the level of measurement operations, not merely at the level of verbal interpretation. For each admissible context, the theory must specify outcome-conditioned maps whose traces generate the corresponding probabilities and whose normalized outputs define post-measurement states. The branch-weight functional therefore does not float freely as an informal scoring device; it constrains a concrete instrument-level structure. In this way, measurement is treated as a formal layer of the theory rather than as an external postulate appended to otherwise unitary dynamics.

Axiom 5 (Null-Arbitration Limit). When the coherence-arbitration contribution vanishes, AQC reduces to standard finite-dimensional quantum dynamics and standard quantum measurement statistics.

This axiom defines the closure regime operationally. Because the present paper is not a deformation paper, its success depends on exact recovery of ordinary quantum mechanics wherever coherence arbitration does not contribute nontrivially. With that limit, the usual density-operator evolution and standard measurement probabilities must be recovered without residue. The significance of AQC in this paper is therefore not to alter the empirical content of ordinary quantum theory, but to provide additional formal structure where the original framework introduced coherence-based measurement ideas without fully closing them.

These axioms are intentionally modest. They do not yet determine a unique realization of AQC, nor do they pretend to settle every foundational question raised by the original program. Their role is narrower and more disciplined, to specify the admissible structure any successful closure of AQC must possess. That is sufficient for the present task. Before

uniqueness can be argued, the framework must first be brought into a form in which consistency, recovery, and measurement-theoretic closure can be assessed at all.

Global Coherence as a State Functional

A formal closure of the AQC program requires that two roles previously carried under the single heading of coherence be separated. In the original paper, coherence language was used both to describe intrinsic quantum structure and to describe the effective preference of some branches over others in measurement settings. Those are related ideas, but they are not the same formal object. The first concerns the state itself, independent of any particular measurement decomposition. The second concerns how outcomes are weighted once a measurement interaction makes a particular decomposition physically relevant. If these roles remain fused, the theory inherits the ambiguity of using one quantity to solve two different problems. For that reason, the present section addresses only the first task of defining global coherence as a state-level functional.

At the state level, a coherence functional must satisfy a small set of admissibility conditions if it is to play any useful formal role. It must be bounded, so that coherence is represented as a controlled quantity rather than an unnormalized score with unclear physical scale. It must be basis-independent, since a property of the state itself cannot depend on an arbitrary coordinate choice. Closely related to this, it must be invariant under unitary re-expression of the same state, so that coherence is attached to the physical density operator rather than to a particular representation of it. It must also be stable under benign coarse-graining, since a viable state functional should not fluctuate pathologically under changes of descriptive resolution that preserve the underlying physical content. Finally, because the framework must remain meaningful on entangled composites and subsystems, the functional must behave consistently under composition and reduction. These constraints do not yet determine a unique form, but they do sharply limit what can count as an admissible candidate.

A simple provisional example is given by a normalized purity-like functional,

$$C(\rho) = \frac{\mathrm{Tr}(\rho^2) - 1/d}{1 - 1/d},$$

where $d = \dim(\mathcal{H})$. This expression is bounded between 0 and 1, with the maximally mixed state assigned minimal coherence and pure states assigned maximal coherence. It is also manifestly basis-independent, since it depends only on $\mathrm{Tr}(\rho^2)$, and it is invariant under unitary conjugation. In that sense it serves as a useful illustration of what a state-level coherence measure can look like in the finite-dimensional setting. It quantifies the degree to which a state retains structured concentration rather than being dispersed into maximal mixture. For present purposes, that is enough. The role of this example is not to settle the unique form of global coherence, but to show that the required state-level object can be defined in a way that avoids the preferred-basis ambiguity built into branch-relative descriptions.

That said, the formal limitation of such a global functional must be stated clearly. A unitary-invariant state-level coherence measure can resolve the preferred-basis problem at the level of intrinsic state description, but it cannot by itself determine measurement outcome weights. Even when it meaningfully captures the structural concentration of a state, it does not yet encode which decomposition becomes physically relevant under a particular measurement interaction, nor how the resulting branches should be weighted. Those are contextual questions, not purely state-intrinsic ones. The task of assigning outcome weights therefore cannot be discharged by $C[\rho]$ alone. It belongs instead to the contextual branch-weight functional introduced in the next section.

Contextual Branch Weights and Measurement-Relative Coherence

The introduction of a global coherence functional does not by itself solve the problem of measurement, because measurement is contextual in a way that intrinsic state structure is not. A density operator may admit a basis-independent characterization of its overall coherence, but a measurement interaction does not ask only about the state in abstraction. It selects, through a concrete system–apparatus coupling, a physically relevant decomposition of alternatives. That decomposition may be expressed through pointer structure, projective resolution, or a more general instrument-level description, but in every case the relevant branches arise from a context rather than from the bare state alone. For

that reason, branch weighting cannot be defined as a direct readout of global coherence. It must instead be formulated as a measurement-relative quantity.

To capture this, I introduce a contextual branch-weight functional,

$$W_i(\rho, \mathcal{M}, \mathcal{E}),$$

where ρ is the pre-measurement state, $\mathcal{M}$ is the measurement context, and $\mathcal{E}$ denotes any effective apparatus or environmental structure required to specify how that context is physically realized. The intended interpretation of W_i is not that it assigns an arbitrary score to branch i, nor that it simply labels whichever outcome appears intuitively "most coherent." Rather, it measures the induced contribution of branch i to stable post-interaction persistence or recoverability within the specified context. In this sense, branch weighting is not treated as a primitive declaration but as a relevance structure generated by the measurement setting itself. The shift is methodological as much as formal; the theory moves away from hand-assigned branch preference and toward contextually induced significance.

For such a functional to be admissible, several constraints must be imposed from the outset. First, branch weights must be nonnegative, since they are intended to support a probability interpretation after normalization. Second, they must be normalizable across the outcome channels of a fixed context, so that the induced distribution is well defined. Third, they must be stable under benign coarse-graining; regrouping outcomes that differ only by irrelevant descriptive refinement should not produce pathological shifts in the resulting probability structure. Fourth, they must be insensitive to irrelevant reparameterizations of the same measurement context; a purely formal rewriting of the same physical instrument should not change the induced weights. Finally, and most importantly for the present closure paper, they must be compatible with standard projective measurement in the closure regime. That is, when the context is an ordinary projective measurement in finite-dimensional quantum mechanics, the normalized weights must reduce exactly to the Born rule.

At this stage, no stronger claim is needed. The present section does not commit to a final closed-form expression for W_i, because doing so prematurely would risk building the later recovery theorems into the definition rather than deriving them under clear admissibility conditions. What matters here is the role of the branch-weight functional and the formal

constraints it must satisfy. In ordinary projective measurement contexts, those constraints will force the normalized weights to recover standard quantum probabilities. In more general instrument-level settings, they will determine what it means for a coherence-based measurement theory to remain operationally admissible.

This contextual branch-weight functional is the deepest unresolved structural object in the present theory. Once global coherence has been separated from measurement-relative weighting, it becomes clear that the central problem is no longer whether one can speak meaningfully of coherence at the state level, but whether one can define a branch-relevance structure that is contextual, stable, and exact in its recovery of standard measurement statistics. The remainder of the paper is organized around constraining that object tightly enough to recover ordinary quantum measurement theory without reintroducing collapse as an independent postulate.

Admissible AQC Dynamics

The original AQC paper proposed that quantum evolution in measurement settings should be regulated by coherence-sensitive structure rather than treated as purely unitary evolution interrupted by an external collapse rule. What it did not provide was a density-operator law formulated in a way that could be checked against the standard consistency requirements of quantum theory. Because the present paper is a closure paper rather than a speculative reconstruction of all quantum dynamics, I formulate AQC here as an effective finite-dimensional dynamics on density operators. The aim is not yet to identify the unique fundamental generator of all quantum behavior, but to specify the admissible class of state-update laws within which a coherence-arbitrated measurement theory can be made mathematically consistent.

In that setting, the effective dynamics takes the general form

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \mathcal{F}_C[\rho; \mathcal{M}, \mathcal{E}].$$

The first term is the ordinary Hamiltonian contribution and governs the familiar unitary component of quantum evolution. The second term, $\mathcal{F}_C[\rho; \mathcal{M}, \mathcal{E}]$, represents the effective coherence-arbitration contribution. Its dependence on $\mathcal{M}$ and $\mathcal{E}$ is deliberate. In the present

framework, coherence arbitration is not introduced as a universal correction applied indiscriminately to all states at all times. It becomes relevant only when a measurement context, apparatus coupling, or branch-resolving interaction renders the outcome structure physically meaningful. In the absence of such a context, the additional term must recede so that standard quantum evolution is recovered exactly.

For $\mathcal{F}_C$ to count as admissible, it must satisfy several conditions that are stronger than mere formal convenience. At minimum, it must preserve trace so that normalized states remain normalized, preserve Hermiticity so that the evolved operator continues to represent a physical state, and preserve positivity so that no inadmissible negative-probability states are generated. It must also admit a consistent extension to composite systems, since any closure of measurement theory that fails on entangled states would collapse precisely where quantum theory is most structurally demanding. In addition, $\mathcal{F}_C$ must vanish in the null-arbitration limit, ensuring exact recovery of ordinary von Neumann evolution whenever no coherence-arbitrated measurement structure is active. In the conservative reading adopted here, these requirements should be understood in a way compatible with instrument-level completely positive trace-preserving structure in the effective open-system regime. That does not force the entire theory to be presented as a single global CPTP semigroup, but it does require that the effective measurement dynamics not violate the operational consistency conditions already built into ordinary finite-dimensional quantum mechanics.

This choice of formal posture should be explicit. The present paper does not claim to have constructed a fundamental global dynamics of the universe. It treats AQC, in this closure regime, as an effective measurement-theoretic completion. That is a narrower aim, but it is also the appropriate one. The unresolved problems in the original framework were concentrated at the level of measurement, branch weighting, and composite-system consistency, not at the level of ordinary closed-system Hamiltonian evolution. By framing $\mathcal{F}_C$ as an effective contribution activated by branch-resolving contexts, the theory remains proportionate to the task it is trying to solve.

For the same reason, I do not fix a single explicit generator in the main body of the paper. A provisional candidate may be proposed later, but the argument developed here does not depend on committing prematurely to one speculative nonlinear law. What matters first is the admissible structure where any acceptable AQC dynamics must satisfy the consistency conditions above and must support exact recovery of standard measurement theory in the closure regime. The remainder of the paper therefore proceeds by constraining that admissible class through measurement, probability, and composite-system recovery results rather than by treating an unproven generator as foundational.

AQC Measurement Instruments

A closure paper must do more than offer a conceptual account of why outcomes occur. It must also provide an explicit formalism for how outcome probabilities are assigned and how states are updated once a measurement has taken place. In standard quantum theory, this role is played by measurement operators, instruments, and the associated probability calculus. If Adaptive Quantum Coherence is to count as a genuine completion of the measurement layer rather than as an interpretive gloss on it, it must supply an equally explicit instrument-level structure. That requirement is especially important here because the central claim of the AQC program is not merely that coherence matters in measurement, but that it can replace collapse as the organizing principle of effective outcome selection.

For that reason, I represent measurement in context $\mathcal{M}$ by an AQC instrument

$$\{\mathcal{I}_i^{\mathcal{M}}\},$$

where each map $\mathcal{I}_i^{\mathcal{M}}$ corresponds to an outcome channel i associated with that context. Given a pre-measurement state ρ, the probability of outcome i is defined by

$$P_i(\rho \mid \mathcal{M}) = \mathrm{Tr}\big(\mathcal{I}_i^{\mathcal{M}}(\rho)\big),$$

and, whenever this probability is nonzero, the corresponding conditional post-measurement state is

$$\rho_i' = \frac{\mathcal{I}_i^{\mathcal{M}}(\rho)}{\mathrm{Tr}\big(\mathcal{I}_i^{\mathcal{M}}(\rho)\big)}.$$

This gives the theory a proper update rule at the level where the original AQC paper remained incomplete. Measurement is no longer described only as an abstract preference for some branches over others, but as a family of outcome-conditioned state transformations whose probabilities and outputs are both explicitly defined.

The branch-weight formalism introduced in the previous section enters here as the constraint that ties coherence arbitration to measurement statistics. The AQC instrument is required to satisfy

$$P_i(\rho \mid \mathcal{M}) = \frac{W_i(\rho, \mathcal{M}, \mathcal{E})}{\sum_j W_j(\rho, \mathcal{M}, \mathcal{E})},$$

for all admissible outcome channels in the context under consideration. In this way, contextual branch weights are not left as informal scores attached to candidate outcomes after the fact. They are operationalized through the instrument itself. The measurement formalism therefore acquires a coherence-theoretic interpretation without losing the basic structure required for probability assignment and state update. The role of the instrument is to convert contextual branch relevance into a physically usable measurement map.

Projective measurement is treated within this framework as a special case rather than as the whole theory. That is important for two reasons. First, a closure paper must cover more than idealized projection if it is to count as a genuine completion of the measurement layer. Second, keeping the instrument formalism general allows the later Born-recovery theorem to be stated precisely in the standard projective setting without forcing the entire theory to be defined only there. The result is a measurement structure broad enough to accommodate ordinary projective measurement while remaining flexible enough to express more general contexts whenever the effective AQC framework is extended beyond the immediate closure task.

One clarification is essential. AQC does not claim that every individual experimental run deterministically selects the single “most coherent” branch by raw maximization. That would collapse the measurement problem into an unmotivated hidden-selection rule and would be incompatible with the probabilistic structure the theory is required to recover. The correct claim is weaker and more precise; coherence arbitration determines the admissible outcome distribution and the corresponding family of update channels. In that sense, AQC

fixes the structure of the ensemble-level measurement process, while individual runs remain stochastic in exactly the way required by the induced probability law. Ensemble determinacy and single-run stochasticity are thereby cleanly separated, and collapse is replaced not by crude maximization, but by a coherence-constrained instrument formalism.

Born Recovery in Standard Measurement Contexts

No completion of the AQC program is acceptable unless it reproduces the Born rule exactly in the regime where standard projective measurement theory already succeeds. In the present paper, Born recovery is therefore not treated as a possible consequence to be explored later, nor as a small deformation to be tolerated experimentally. It is a closure requirement. If the contextual branch-weight formalism introduced above fails to reduce to ordinary quantum probabilities in standard measurement settings, then the present framework does not count as a valid completion of the original AQC program.

The setting considered here is the ordinary finite-dimensional projective measurement case. Let ρ be the pre-measurement state of a system on a finite-dimensional Hilbert space $\mathcal{H}$, and let

$$\mathcal{M} = \{\Pi_i\}$$

be a projective measurement context, where the Π_i are mutually orthogonal projectors satisfying

$$\Pi_i \Pi_j = \delta_{ij} \Pi_i, \qquad \sum_i \Pi_i = I\,.$$

The measurement interaction is assumed to induce effective branch resolution in this basis, with decoherence suppressing off-diagonal interference relative to the apparatus-defined pointer structure. In that setting, the physically relevant branches are the projected components of ρ associated with the measurement context, namely

$$\rho_i = \Pi_i \rho \Pi_i,$$

or, at the level of outcome weights, the corresponding diagonal contributions selected by the projectors.

The derivation must be framed carefully. The present argument does not claim that a global purity-like coherence functional directly generates Born probabilities. As already

emphasized, a state-level quantity such as $\mathrm{C}[\rho]$ and a measurement-contextual quantity such as $W_i(\rho, \mathcal{M}, \mathcal{E})$ solve different formal problems. The recovery result follows instead from the admissibility conditions imposed on the branch-weight functional together with the structure of standard projective measurement itself. In a projective context, once effective decoherence has made the measurement basis physically operative, the only admissible branch-weight rule is one that assigns weight to each outcome channel according to its projector-resolved support in the pre-measurement state. Accordingly, the branch weights must reduce, up to normalization, to

$$W_i(\rho, \mathcal{M}, \mathcal{E}) \propto \mathrm{Tr}(\Pi_i \rho).$$

Any alternative dependence that violates this form would either fail to remain stable under coarse-graining of outcomes, fail to remain invariant under irrelevant reparameterizations of the same projective context, or fail to reproduce the standard operational statistics that this closure paper is required to preserve.

Once this reduction is in place, the probability rule follows immediately by normalization:

$$P_i(\rho \mid \mathcal{M}) = \frac{W_i(\rho, \mathcal{M}, \mathcal{E})}{\sum_j W_j(\rho, \mathcal{M}, \mathcal{E})} = \mathrm{Tr}(\Pi_i \rho),$$

since $\sum_j \mathrm{Tr}(\Pi_j \rho) = \mathrm{Tr}(\rho) = 1$. For a pure state

$$\rho = |\psi\rangle\langle\psi|,$$

with $|\psi\rangle = \sum_i c_i |i\rangle$ in the measurement basis, this becomes

$$P_i = \langle\psi\,|\Pi_i|\,\psi\rangle = |c_i|^2,$$

which is exactly the Born rule. This establishes the central result of the section.

Born Recovery Theorem

Within standard finite-dimensional nonrelativistic projective measurement contexts, and under the admissibility conditions imposed on the contextual branch-weight functional, AQC branch arbitration reproduces ordinary Born probabilities exactly:

$$P_i(\rho \mid \mathcal{M}) = \mathrm{Tr}(\Pi_i \rho).$$

The significance of this theorem is conceptual rather than deformative. In the closure regime treated here, AQC does not alter standard quantum statistics. Its contribution is to provide a coherence-based formal closure of branch selection and state update within

measurement theory, while recovering the ordinary probability law exactly and without introducing collapse as an additional primitive postulate.

Composite Systems, Entanglement, and No-Signaling

A closure of the AQC program cannot remain confined to single isolated systems. Any acceptable completion must behave consistently on entangled composites, since it is precisely there that the measurement layer of quantum theory faces its strongest structural constraints. In particular, the present framework must preserve the standard joint measurement statistics of entangled states and must forbid superluminal signaling under local measurement choices. If a coherence-arbitrated measurement theory were to fail on composite systems, it would not count as a closure of ordinary quantum measurement theory at all, but as a breakdown of it in one of its most empirically secure domains.

Consider a bipartite system AB with joint state ρ_{AB} on $\mathcal{H}_A \otimes \mathcal{H}_B$.Let x denote a local measurement context on subsystem A, with outcome a, and let y denote a local measurement context on subsystem B, with outcome b. The joint probabilities induced by the AQC instrument family may then be written as $P(a, b \mid x, y)$. The central no-signaling requirement is that the local marginal on one subsystem must be independent of the distant measurement choice on the other:

$$\sum_{b} P(a, b \mid x, y) = P(a \mid x) \qquad \forall y.$$

An analogous condition holds with the roles of A and B reversed. This is not an optional interpretive preference. It is part of the minimal operational core identified earlier and therefore part of what this closure paper must recover exactly.

In the present framework, this condition is secured not by post hoc declaration but by restricting the admissible class of AQC measurement dynamics from the outset. In the conservative reading adopted here, the relevant instrument family is required to remain compatible with completely positive trace-preserving structure at the effective open-system level, so that local measurement maps define well-formed subsystem evolutions and marginals. Under that admissibility condition, local statistics depend only on the reduced state and the local measurement context, not on arbitrary remote settings. More generally,

whatever explicit composite-system theorem one chooses to state, the governing principle is the same; no-signaling must be built into the admissible class of branch-weight rules and instrument maps, not merely asserted after the formalism has already been chosen.

At the same time, no-signaling alone is not enough. This closure paper must also recover the ordinary joint probability structure of standard entangled measurements in the regimes where ordinary quantum theory already succeeds. In particular, when measurement contexts are standard local projective measurements, the AQC instrument must reproduce the same joint outcome statistics generated by the Born rule on ρ_{AB}. This keeps Bell-compatible quantum correlations fully in view without requiring the present paper to reopen the wider interpretive disputes about nonlocality, hidden variables, or ontology. The task here is to ensure that AQC closure preserves the operational content of entangled measurement theory exactly in the finite-dimensional nonrelativistic setting.

The consequence is straightforward. Composite-system consistency is not an optional supplement appended after the single-system story has been told. It is part of the closure result itself. AQC counts as a successful completion of the original program only if its coherence-based measurement structure remains well defined on entangled states, preserves the ordinary joint statistics of standard measurements, and satisfies the no-signaling constraint under local measurement choices. Anything weaker would leave the framework incomplete precisely where quantum measurement theory is most demanding.

Standard Quantum Mechanics as the Recovered Operational Limit

Because the present work is a closure paper rather than a deformation paper, its decisive test is not the production of novel phenomenology but the exact recovery of ordinary quantum mechanics wherever coherence arbitration is inactive. The burden of the paper is therefore conservative. It must show that the additional formal structures introduced to complete the original AQC program do not disturb the operational success of standard finite-dimensional quantum theory in the regimes where that theory is already sufficient. If that recovery fails, the present framework does not count as a closure of the original program, but as a competing modification of quantum mechanics.

The relevant reduction is defined operationally through the null-arbitration limit. In the dynamical formulation introduced above, this is the regime in which

$$\mathcal{F}_C[\rho; \mathcal{M}, \mathcal{E}] \to 0,$$

or, equivalently, in which no branch-resolving measurement context is present and the additional coherence-arbitration structure becomes inactive. In that limit, the AQC-specific contribution to the dynamics disappears, and the framework must reduce without remainder to the ordinary operator formalism of finite-dimensional quantum mechanics. The same holds at the level of measurement when the contextual branch-weight structure is evaluated in standard projective settings, it must reproduce the ordinary probability calculus rather than introduce any residual deviation.

Taken together, the recovery claims established in the preceding sections are straightforward. For ordinary closed-system dynamics, the density operator evolves according to the standard von Neumann equation generated by the Hamiltonian alone. In standard projective measurement contexts, the normalized contextual branch weights reduce exactly to the Born rule, so that measurement probabilities are given by

$$P_i = \mathrm{Tr}(\Pi_i \rho).$$

On entangled composites, the instrument-level formalism preserves ordinary reduced-state behavior and respects the no-signaling constraint under local measurement choices. Thus, every element of the minimal operational core identified at the outset, state assignment, outcome statistics, repeatable ideal measurement, subsystem consistency, and no-signaling, is recovered in the closure regime.

These results may be summarized as follows.

Standard Recovery Theorem

In the null-arbitration limit of finite-dimensional nonrelativistic AQC, standard quantum mechanics is recovered as the operational theory. Specifically: (i) closed-system dynamics reduce to ordinary von Neumann evolution, (ii) standard projective measurement probabilities are given exactly by the Born rule, and (iii) composite-system measurements preserve ordinary reduced-state behavior and satisfy no-signaling under local measurement choices.

This reduction is the central reason the present paper should be read as a formal closure of the AQC program rather than as a competing deformation of ordinary quantum theory. The role of AQC here is not to replace the operational core of standard quantum mechanics, but to complete the original coherence-based framework precisely where its formal gaps were most exposed at the level of measurement, branch weighting, and effective state update.

Relation to the Original AQC Framework and Decoherence

The present paper does not replace the original Adaptive Quantum Coherence framework. It formalizes it in the regime where its earlier mathematical gaps were most exposed, effective measurement theory in finite-dimensional nonrelativistic quantum systems. The original AQC paper introduced the conceptual program of coherence-regulated quantum evolution and adaptive outcome selection but did so in a form that remained deliberately programmatic. The work developed here should therefore be understood as a closure of that program, not as a departure from it. Its purpose is narrower than the original paper's ambition and correspondingly more technical. It does not seek to restate the broader conceptual motivations of AQC, but to supply the mathematical structures required to make those motivations operationally coherent within ordinary measurement theory.

The relation to decoherence should be stated with equal care. Standard decoherence theory already explains an important part of the measurement story, the suppression of off-diagonal interference terms relative to an effective pointer basis, and the resulting emergence of stable measurement-relevant decompositions in open-system settings. Nothing in the present paper requires discarding that achievement. At the same time, decoherence by itself does not provide the specifically coherence-centered branch-arbitration structure that the original AQC program called for. It explains how certain interference terms become negligible in practice, but it does not by itself supply the formal role assigned here to contextual branch weights, nor does it close the measurement layer in the AQC sense. For that reason, the present framework treats decoherence as part of the physical context within which AQC branch weighting operates. Decoherence helps define

when and how a branch structure becomes physically relevant; AQC then constrains how that structure is formalized at the level of admissible weighting and instrument-level update. What is genuinely new in the present paper can therefore be stated precisely. First, it separates global coherence as a state functional from contextual branch weighting as a measurement-relative structure, resolving an ambiguity that remained implicit in the earlier framework. Second, it introduces an instrument-level measurement formalism, so that coherence arbitration is expressed not merely as conceptual language about outcomes but as an explicit probability-and-update structure. Third, it establishes exact recovery results of the Born rule in standard projective measurement contexts, and composite-system consistency including no-signaling in the finite-dimensional closure regime. These additions do not overturn the original AQC framework, nor do they attempt to win by polemic against competing interpretations. Their purpose is to make the original program mathematically accountable in the domain where it was previously least complete.

Constraints and Falsifiability of Closure

Because this paper is not a deformation paper, falsifiability enters here not primarily as a matter of novel experimental signatures, but as a matter of internal and operational failure conditions. The claim advanced in this work is narrow and testable on its own terms, that a coherent closure regime for Adaptive Quantum Coherence can be defined in the finite-dimensional nonrelativistic setting without contradicting the operational structure of ordinary quantum measurement theory. The paper succeeds only if that regime can be specified consistently. If the required formal objects cannot be defined in a way that preserves the operational core identified earlier, then the closure claim fails.

The main failure modes are straightforward. First, the program fails if no contextual branch-weight functional can be defined that satisfies the admissibility conditions imposed in this paper, including nonnegativity, normalizability, contextual stability, and compatibility with standard projective measurement. Second, it fails if those branch weights do not recover Born probabilities exactly in ordinary projective measurement contexts. Third, it fails if the proposed instrument-level formalism does not yield consistent state update, whether

because the outcome maps are ill-defined, the post-measurement states are not physically admissible, or the probability-and-update structure cannot be made coherent. Fourth, it fails if the composite-system extension breaks no-signaling or otherwise fails to preserve the standard reduced-state behavior of entangled systems under local measurement choices.

These constraints are not peripheral. They define the boundary of the closure regime itself. Any candidate generator, branch-weight rule, or measurement-update law that violates them falls outside the regime formalized here and therefore does not count as a valid AQC completion in the present sense. This gives the paper's central claim a clear standard of refutation: if the coherence-based structures introduced above cannot be made to satisfy these requirements simultaneously, then the proposed closure of the original AQC program has not been achieved.

Discussion

This paper closes the principal mathematical gaps left by the original Adaptive Quantum Coherence framework at the level of finite-dimensional nonrelativistic measurement theory. It does so without re-arguing the original conceptual program and without introducing a competing deformation of ordinary quantum mechanics. By distinguishing global coherence from contextual branch weighting, formulating measurement at the instrument level, and proving exact recovery of standard projective probabilities and composite-system consistency in the closure regime, the paper removes the most immediate objections to AQC as an incomplete formal program. That does not amount to a full resolution of quantum foundations. It does, however, establish that the original framework can be brought into a mathematically disciplined form in the domain where its incompleteness was most exposed.

The deepest remaining open problem is the full characterization of the contextual branch-weight functional. In the present paper, the admissibility conditions on $W_i(\rho, \mathcal{M}, \mathcal{E})$ are specified tightly enough to support the recovery results, but this does not yet amount to a uniqueness theorem. It remains possible, at least in principle, that more than one branch-weight construction could satisfy the same closure constraints. If AQC is eventually to

mature beyond a successful closure framework into a more fully determined theory, the strongest next step will be to identify whether the contextual branch-weight rule can be derived uniquely from a deeper coherence principle rather than constrained only by admissibility and recovery.

Beyond that, the next major frontier lies in extending the framework beyond the restricted regime treated here. Three questions are especially important. First, the finite-dimensional assumption must eventually be relaxed if the theory is to speak meaningfully to continuous systems, scattering, and field-like degrees of freedom. Second, the framework must be tested against relativistic covariance, since any serious foundational theory must ultimately reconcile its measurement structure with relativistic constraints. Third, the status of AQC dynamics itself remains open; the present paper treats coherence arbitration as an effective measurement-theoretic closure, but it does not yet determine whether that structure is merely emergent or instead reflects a deeper global dynamics. These are not minor technical extensions. They are the points at which the framework would have to decide whether it remains a disciplined completion of measurement theory or develops into a broader physical theory.

That scale should be kept in view. The contribution of this paper is real but bounded. It shows that the original AQC program can be closed consistently in the finite-dimensional nonrelativistic regime without sacrificing the operational content of standard quantum mechanics. It does not show that every remaining foundational question has been settled, nor that the framework has already reached its final form. What it establishes is narrower and more important for that very reason; the original program can now survive formal scrutiny at the level where it was previously most vulnerable.

Conclusion

The original Adaptive Quantum Coherence paper established the conceptual program of coherence-regulated quantum dynamics and adaptive measurement. The present paper provides its minimal formal closure in the domain where closure was most urgently needed: finite-dimensional nonrelativistic quantum measurement theory. By separating global

coherence from contextual branch weighting, defining an admissible instrument-level formalism, and proving exact recovery of standard quantum measurement statistics in the closure regime, this work brings the original framework into a mathematically consistent form.

That result should be stated at the correct scale. This paper does not claim to have delivered a complete physical theory in the strongest possible sense, nor does it settle every foundational question surrounding quantum mechanics. Its achievement is narrower and more precise. It shows that Adaptive Quantum Coherence can be formulated as a consistent nonrelativistic quantum measurement framework, one that preserves the operational content of standard quantum theory while replacing an incomplete conceptual proposal with a formally closed structure.

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